**2008 IMSA Junior High Math Competition**

8th Grade Team Contest

1. Emily begins reading *Crime and Punishment* at 7 pm. If she reads 5 pages in 9 minutes, how many pages has she read by 7:45 that evening, assuming she reads at a constant rate?
2. Anne likes to bake cupcakes. On weekdays she bakes 12 cupcakes every day. On weekends, she bakes 24. How many cupcakes does she bake in a week?

3. Eight people split the cost of 3 equally priced pizzas. However, after they split the cost, they decide to buy one more for the same price. If each person ends up paying $1.33 more than what they had planned to pay previously, how much did each pizza cost?

4. How many diagonals does a 10 sided regular polygon have?

5. Sam is 5 years younger than his sister Lina. If in 4 years Sam is 2/3 of Lina’s age, how old is he now?

6. What is the greatest prime factor of 2006+2007+2008?

7. Express the repeating decimal 7.89898989898989… as a fraction reduced to lowest terms.

8. Simplify: 

9. Find the greatest common factor between 2!+3!+4! and 5!+6!+7!

10. How many distinct triangles with integer side lengths can be formed with perimeter 13?

11. How many digits does  have?

12. An ant is located in a corner of a rectangular room with dimensions 10 ft x 30 ft and 7 ft in height. The ant wishes to crawl from a corner on the ground to the opposite corner on the ceiling. How long is the shortest path it can take?

13. What is 15% of 25% of the product of 15 and 25 plus 25% of 15 % of the product of 25 and 15?

14. What is the volume of a regular tetrahedron with side length 4?

15. If  and *a, b, c, d* and *e* are positive integers such that no pair has a common factor larger than 1, find the ordered set (*a, b, c, d, e*).

16. Point A is at (1,2), Point B is at (7,3), Point C is at (6,5), and Point D is at (2,9). What is the area of Quadrilateral ABCD?

17. Nunu is always in one of three rooms—numbered 1, 2, and 3—which are arranged in a row in order of their number. However, every 5 minutes she may move to an adjacent room. There is a 20% probability she will move to a room with a lower number (or stay put for one turn if she is in room 1) and an 80% probability she will move to a room with a higher number (or stay put if she is in room 3). If she begins in room 1, what is the probability that she is in room 1 again after 21 minutes have passed?

18. Find the value of 2!+0!+0!+8!

19. Sharon looks at the clock. The hour hand and the minute hand form a 71 degree angle. Thirty minutes later, what are the possibilities of the non-reflexive angles (less than 180 degrees) formed by the hands of the clock, rounded to the nearest degree?

20. A decreasing number is a number such that each digit is less than the one to its left. 2, 21, 430 and 921 are decreasing numbers. 15, 544 and 914 are not decreasing numbers. How many decreasing numbers are there?

**2009 IMSA Junior High Math Competition**

8th Grade Team Contest

1. A triangle has side-lengths 6, 8, and 10. What is its area?
2. Puccini’s libretto is 259 pages long. If he reads 7 pages each day, how many days will it take him to finish reading the libretto?
3. Rusalka counts 6 lily-pads and 4 logs floating in the pond. If three frogs can fit on a lily-pad and 13 can fit on a log, what is the maximum possible number of frogs?
4. Musetta owns thirteen pairs of shoes, eleven skirts, eight blouses, and four hats. How many ways can she pick out an outfit if she *must* wear shoes, a skirt, and a blouse, but a hat is optional?
5. Magda is thinking of a two-digit number. She says, “If you add my number to 51, the sum will be divisible by 17.” If I randomly choose one of the numbers that satisfies the condition, what is the probability that I guessed her number correctly?
6. Find all numbers *n* for which: .
7. A circle with area 1 and a square with area 1 are drawn on a piece of paper such that they share the same center. What is the area of the region inside both the circle and the square plus the region inside the square that is *not* inside the circle?
8. What is the units digit of (10!)2009?
9. Mr. Barzun can type five pages in three minutes and Miss Hewhay can type twenty-six pages in six minutes. How long will it take them working together to type 64 pages if each works at a constant rate? Express your answer in hours and as a mixed number.
10. Alberich sells his ring to Wotan, for 110% of what the raw materials cost. Then Wotan marks it up to twice what he paid Alberich and sells it to Freia. She pays $242.00. How much did the raw materials cost Alberich (in dollars)?
11. Zimmermann draws a regular 81-sided polygon on a piece of paper. What is the degree measure of an exterior angle?
12. At a party of thirty people, no one knows anyone else. How many face-to-face meetings must take place for everyone to know everyone else [a face-to-face meeting is a meeting between two people]?
13. The volume of a cube is five times its surface area. What is the length of an edge of the cube?
14. How many distinct ways are there to arrange the letters in ZEFFIRELLI?

15. Find the smallest n such that n! ends in 290 zeroes.

16. Gilda has somewhere between 1000 and 2000 pieces of paper he's trying to divide into piles of the same size (but not all in one pile or piles of one sheet each). He tries 2, 3, 4, 5, 6, 7, and 8 piles but ends up with one sheet left over each time. How many piles does he need?

17. If 4 is added to the numerator of a fraction and 10 is added to the denominator, the value of the fraction remains unchanged. Find the value of the fraction.

18. A rectangle has perimeter 10 and diagonal. What is its area?

19. A parallelogram has 3 of its vertices at (1, 2), (3,8), and (4, 1). Compute the sum of the possible x-coordinates for the 4th vertex.

20. If a and b represent single digits, find all ordered pairs (a,b) such that 23199249ab is divisible by 24.

**2010 IMSA Junior High Math Competition**

8th Grade Team Contest

1. If a blind man has a sock drawer with 70 black socks, 70 white socks, and 70 brown socks, how many socks must he pick to guarantee that he has a matching pair?
2. Find the angle between the hour and minute hand of a clock at time 12:47. (Give the exact angle less than ).
3. Samuel has quarters and nickels in his pockets, for a total of 20 coins all together. The value all this change totals $2.60. Find the number of quarters in his pocket.
4. Alex’s empty swimming pool will hold 15,000 gallons of water when full. It will be ﬁlled by 5 hoses, each of which supplies 3 gallons of water per minute. How many hours will it take to fill this pool?
5. The sum of three positive integers is 37. If one of these integers is 9, what is the greatest possible value of the product of the other two integers?
6. A square with an area of 64 has the same side length as a regular hexagon ( a polygon with six sides of equal length). What is the area of the hexagon?
7. Two angles of an isosceles triangle measure and. What is the sum of all possible values of *x*?
8. How many whole numbers between 1 and 1000 inclusive do not contain the digit 5 or 8?
9. Samantha is preparing to have 37.5% off sale at her clothing store. However, being a sneaky merchant, she wants to increase her regular prices right before the sale so that the sale price is the same as the original price before any price changes are made. By what percent must she raise the prices before the sale to accomplish her goal?
10. Simplify the following complex fraction:



1. What is the smallest positive number can be subtracted from both the numerator and the denominator of 29/34 so the resulting fraction will be equivalent to 4/5?
2. Suppose *a*, *b,* and *c* are positive integers with 3*a*5*b*2*c*=60750. What is the value of *abc*?
3. The clover below is formed by a square and 4 semicircles. If the side of the square is 6, find the area of the 4 “petals” of the clover.

1. Dr. Condie is taking care of 5 unique babies, and it is play time! If he has 2 distinct playpens, and plans to put all the babies in playpens with at least 1 baby is each pen, in how many different arrangements can he place the babies?
2. If two standard dice are rolled, what is the probability the product of the two numbers rolled is 20 or higher?
3. What is the smallest integer you must multiply 4320 by in order to have a perfect square number?
4. Two numbers have a sum of 528. One of the numbers has a zero as one of its digits. If you remove the zero from this number, then it is equal to the other number. What is the larger of the two numbers that sum to 528?
5. Take any right triangle form a new triangle by connecting the midpoints of the three sides of the right triangle. Repeat this process with the new triangle to form a third triangle. Find the ratio of the area of the smallest triangle to the area of the original right triangle.
6. There is a cylindrical can of 3 spherical tennis balls, each with radius 4cm, touching each other and the sides of the cylinder. How much empty space is there within the
container with the balls in it?
7. In the cryptarithm MEMO + FROM = HOMER, every letter stands for a distinct digit between 0 and 9. What is the value of M?

**2011 IMSA Junior High Math Competition**

1. An old woman carrying a basket of strawberries dropped her basket accidentally. The berries rolled out, but a stranger stopped and helped her pick them up.
“How many strawberries did you have?" the stranger asked.
“I don't quite remember. Well, when I tried to divide them into two baskets, there was one strawberry left over. When I divided them into three baskets, there were two left over, and there were three strawberries left over when I tried to divide them into four baskets. Four strawberries were left over when I divided them into five baskets," she answered. What is the smallest number of berries that she could have started with in her basket?
2. Find the smallest perfect square that is a multiple of 140.
3. If is a factor of (230!), what is the largest possible value of *n*?

1. What is the coefficient of the $x^{2}y^{5}$ term in the expansion of $\left(-2x+y\right)^{7}$?
2. The greatest common divisor of 360, 900, and $x $is 90. If $600<x<800$, what is $x$?
3. It takes Katie 30 minutes to drive from her house to Corinne’s house at 65 miles per hour. If Corinne is driving at 40 miles per hour, how many minutes will it take her to get from her house to Katie’s house?
4. Andrew is randomly drawing from a deck of cards (which contains 52 cards with 13 of each suit) without replacement. What is the probability that Andrew draws the four aces in any order?
5. If a salad calls for 2 lettuce leaves per 3 slices of cucumbers, 4 slices of cucumbers per 5 croutons, 6 croutons per 7 cherry tomatoes, and 8 cherry tomatoes per 9 spinach leaves, what is the ratio of lettuce leaves to spinach leaves?
6. If $a, b, c and d$ are all distinct integers and $a>b, c>b, d>a, and d>c$, what is the sum of all the possibilities for *c* provided that d is 18, a is 13, and b is 10?
7. Out of 50 marbles, there are 10 pink marbles, 14 green marbles, 18 blue marbles, and some number of yellow marbles. Martin wins if he can draw with replacement one marble of each color in any order (so one pink marble, one green, one blue, and one yellow). He loses if he draws a marble that matches the color of one he has already drawn. What is the probability that he loses on his second draw?
8. If $x^{2}+y^{2}=40$ and $xy=11$, then what does $\left(x+y\right)^{2}$ equal?
9. Find the area of a regular hexagon with sides of length 10 inches.
10. Dr. Condie is a coin collecting fanatic. He happens to like the New York and Massachusetts State quarters the best, and has 29 and 11 of them, respectively. All of his New York quarters are kept heads up, while all of his Massachusetts quarters are kept heads down. Dr. Keyton decides to run in and flip over 20 of Dr. Condie’s quarters randomly. Find the expected number of Dr. Condie’s quarters that are heads up when Dr. Keyton is finished.
11. How many integers $x$satisfy the following inequality?
$$\left(x-2006\right)\left(x-2004\right)\left(x-2002\right)…(x-4)\leq 0$$
12. Dr. Prince bought a dozen cookies. He wanted at least one each of M&M, sugar, and walnut. Find the number of different combinations of cookies that he could have bought.
13. Find the number of ordered pairs of positive integers (a, b, c, d) that satisfy the following

equation:

$$a+b+c+d=13$$

1. When a right triangle is rotated about one leg, a cone with volume 4800π is formed. When the triangle is rotated about its other leg, a cone with volume 1080π is formed. What is the hypotenuse of the triangle?
2. What is the smallest integer $n$ such that$ n!$ ends in at least 100 zeros?
3. Find the last two digits of$12^{12}^{12}$.
4. Three regular, six-sided dice are rolled successively. What is the probability that they are in strictly decreasing order?

 2012 IMSA Junior High Mathematics Competition

1. The weatherman predicted that there is a 40% chance of rain tomorrow. What is the probability that it will not rain tomorrow? Express your answer as a percent.
2. What is the area of the region bound by the lines x = 0, y = 0, and y = -8x+4?
3. Abhi is playing hot-shot basketball. His probability of making any free throw (1 point) is 2/5, and his probability of making any 3-pointer is 1/10. Suppose that in one hot-shot game, Abhi decides to shoot 6 free throws and 4 3-pointers. What is his expected score? Express your answer as an exact decimal.
4. James, Kevin, and Lael are three members of a 12-member English class. Their teacher wants to divide the class into two groups of six. Let x be the number of ways that the teacher can do this, if James, Kevin, and Lael must all be in the same group. Find the sum of the digits of x.
5. Find the sum of all prime numbers between 42 and 74.
6. How many circles of radius 1 may be placed without overlap within a rectangle of dimensions $7 ×(2\sqrt{3}+2)$?
7. Find the units digit of 72012.
8. Rishi and Srisha are playing a chess match. The first player to win three games wins the match. Rishi has a 3/4 chance of winning any game, while Srisha has a 1/4 chance. Assuming there are no tied games, what is the probability that Rishi wins his chess match against Srisha? Express your answer as a common fraction.
9. You have a semicircle with diameter 8 inches. You decide to cut out two smaller semicircles, each with diameter 4 inches, from the semicircle, with the smaller semicircles’ diameters lying on the diameter of the original semicircle. Find the perimeter of the resulting cut-out shape. Express your answer as a decimal rounded to the nearest hundredth.
10. How many lattice points lie inside or on the circle of radius 3 centered at the origin? (A *lattice point* is a point with both coordinates as integers).
11. Let # be the function defined by $a\#b=a^{3}+b$. Given $x\#488=10^{3}$, find x.
12. Circular arcs are drawn on the sides of an equilateral triangle with side 10 cm. Each arc has its center at the vertex of the triangle it does not intersect. Find the perimeter of the entire figure formed by the arcs. Express your answer in terms of pi.
13. How many numbers leave a remainder of 12 when 1337 is divided by them?
14. Maggie has 20 identical pieces of Hershey’s Kisses. In how many ways can she distribute the candy to 6 of her friends, if her best friend, Evan, must receive at least 2 pieces, and every one of the 6 friends must have at least one piece?
15. Adam can mow a lawn in 3 hours, while Ben can do it in 6 hours, and Carl can do it in 4 hours. If all 3 men work together to mow the lawn, how long will it take them to do so, if Carl gets injured after mowing for 1 hour and cannot continue afterwards? Express your answer as a decimal.
16. Antonio, Wolfgang, Joseph, Johannes, Leonard, Igor, and Ludwig are sitting at a round table, with seats numbered 1 through 7. How many ways can they be seated, if Wolfgang and Ludwig insist on sitting next to each other?
17. If, find .
18. Evaluate $1^{3}+2^{3}+3^{3}+ …+ n^{3}$ if n=10.
19. If $x=\sqrt{2+\sqrt{2+\sqrt{2+…}}}$, solve for the value of *x*.
20. Each square of a 4 x 4 grid is filled with a 1, 2, 3, or 4. Define the four *neighbors* of a square *S* to be the squares directly above, to the right, to the left, and below *S*. If a square is located in the top row, its upper neighbor wraps around to the bottom row (like in the figure below, B, D, C, and E are all neighbors of A).

|  |  |  |  |
| --- | --- | --- | --- |
| A | B |  | D |
| C |  |  |  |
|  |  |  |  |
| E |  |  |  |

Define $f(a,b)$ as the number of unordered pairs of neighboring squares such that one contains $a$ and the other contains $b$. It is given that:

$$f\left(4, 1\right)=6$$

$$f\left(4, 2\right)=3$$

$$ f\left(4, 3\right)=9$$

$$f\left(4, 4\right)=3$$

Find the number of squares containing 4.